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# FM SPECTRAL MODELING AND FDM/FM SIMULATION PROGRAMS

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TABLE OF CONTENTS

ABSTRACT . . . . .	vi
SECTION 1 GENERAL INTRODUCTION . . . . .	1
SECTION 2 FM SPECTRAL MODELING AND GAUSSIAN REPRESENTATIONS. . . . .	3
GAUSSIAN SPECTRAL APPROXIMATION PRINCIPLES . . . . .	3
Analysis of the Series Expansion Representation. . . . .	4
Analysis of the Gaussian Spectral Approximation. . . . .	7
SECTION 3 RECTANGLE CONVOLUTION PROGRAM FOR FM SPECTRUM SIMULATION . . . . .	11
RECTANGLE CONVOLUTION PROGRAM PRINCIPLES . . . . .	11
RECTANGLE CONVOLUTION PROGRAM RESULTS. . . . .	14
SECTION 4 GENERALIZED FM SPECTRUM GENERATION PROGRAM . . . . .	36
DISCRETE FOURIER TRANSFORMS. . . . .	38
NUMBER OF SAMPLES. . . . .	39
BUTTERWORTH BASEBAND SPECTRUM SIMULATION . . . . .	43
PREEMPHASIS AND FM/PM CONVERSION SIMULATIONS . . . . .	50
FDM/FM SPECTRAL SIMULATION RESULTS . . . . .	
SECTION 5 SUMMARY AND DISCUSSION OF RESULTS. . . . .	66
REFERENCES. . . . .	71
APPENDIX A FM BANDWIDTH AND POWER DISTORTION . . . . .	72
APPENDIX B COMPARISON OF FDM/FM SPECTRAL SIMULATIONS . . . . .	77

LIST OF TABLES

Table	
1	Significant Terms (Values of n) vs RMS Phase Deviation ( $\beta$ ) for Various Power Percentages . . . . . 6
2	Residual Carrier Magnitudes in FM Spectra . . . . . 68

TABLE OF CONTENTS (CONTINUED)

LIST OF FIGURES

Figure	1	The Gaussian Envelope Approximation to the Poisson Weighting Coefficients . . . . .	8
	2	Decomposition of $F_n(f)$ into $F_n(k)$ (f) segments . . . . .	12
	3(a)	Sum of N=5 Weighted Convolutions for $\beta=0.2$ . . . . .	15
	3(b)	Sum of N=5 Weighted Convolutions for $\beta=0.5$ . . . . .	15
	3(c)	Sum of N=5 Weighted Convolutions for $\beta=0.7$ . . . . .	16
	3(d)	Sum of N=5 Weighted Convolutions for $\beta=1.0$ . . . . .	16
	3(e)	Sum of N=9 Weighted Convolutions for $\beta=2.0$ . . . . .	17
	3(f)	Sum of N=17 Weighted Convolutions for $\beta=3.0$ . . . . .	17
	4(a)	Case of N=5 Weighted Convolutions for $\beta=0.2$ . . . . .	18
	4(b)	Case of N=5 Weighted Convolutions for $\beta=0.5$ . . . . .	18
	4(c)	Case of N=5 Weighted Convolutions for $\beta=0.7$ . . . . .	19
	4(d)	Case of N=5 Weighted Convolutions for $\beta=1.0$ . . . . .	19
	4(e)	Case of N=9 Weighted Convolutions for $\beta=2.0$ . . . . .	20
	4(f)	Case of N=17 Weighted Convolutions for $\beta=3.0$ . . . . .	20
	5(a)	Sum of Various Convolutions for $\beta=2.0$ . . . . .	21
	5(b)	Sum of Various Convolutions for $\beta=3.0$ . . . . .	21
	6(a)	Case of Various Convolutions for $\beta=2.0$ . . . . .	22
	6(b)	Case of Various Convolutions for $\beta=3.0$ . . . . .	22
	7(a)	Sum of N=5 Weighted Convolutions for $\beta=1.0$ . . . . .	24
	7(b)	Sum of N=6 Weighted Convolutions for $\beta=1.1$ . . . . .	24
	7(c)	Sum of N=6 Weighted Convolutions for $\beta=1.2$ . . . . .	25
	7(d)	Sum of N=6 Weighted Convolutions for $\beta=1.3$ . . . . .	25
	7(e)	Sum of N=6 Weighted Convolutions for $\beta=1.4$ . . . . .	26
	7(f)	Sum of N=8 Weighted Convolutions for $\beta=1.5$ . . . . .	26
	7(g)	Sum of N=8 Weighted Convolutions for $\beta=1.6$ . . . . .	27
	7(h)	Sum of N=8 Weighted Convolutions for $\beta=1.7$ . . . . .	27
	7(i)	Sum of N=9 Weighted Convolutions for $\beta=1.8$ . . . . .	28
	7(j)	Sum of N=9 Weighted Convolutions for $\beta=1.9$ . . . . .	28
	7(k)	Sum of N=9 Weighted Convolutions for $\beta=2.0$ . . . . .	29
	7(l)	Sum of N=17 Weighted Convolutions for $\beta=3.0$ . . . . .	29
	8(a)	Case of N=5 Weighted Convolutions for $\beta=1.0$ . . . . .	30
	8(b)	Case of N=6 Weighted Convolutions for $\beta=1.1$ . . . . .	30
	8(c)	Case of N=6 Weighted Convolutions for $\beta=1.2$ . . . . .	31
	8(d)	Case of N=6 Weighted Convolutions for $\beta=1.3$ . . . . .	31
	8(e)	Case of N=6 Weighted Convolutions for $\beta=1.4$ . . . . .	32
	8(f)	Case of N=8 Weighted Convolutions for $\beta=1.5$ . . . . .	32
	8(g)	Case of N=8 Weighted Convolutions for $\beta=1.6$ . . . . .	33
	8(h)	Case of N=8 Weighted Convolutions for $\beta=1.7$ . . . . .	33
	8(i)	Case of N=9 Weighted Convolutions for $\beta=1.8$ . . . . .	34
	8(j)	Case of N=9 Weighted Convolutions for $\beta=1.9$ . . . . .	34
	8(k)	Case of N=9 Weighted Convolutions for $\beta=2.0$ . . . . .	35
	8(l)	Case of N=17 Weighted Convolutions for $\beta=3.0$ . . . . .	35
	9	Generalized FM Spectrum Generation. . . . .	37
	10(a)	Inverse DFT Realization via Direct DFT. . . . .	40
	10(b)	Inverse DFT Realization for Real Even Symmetry. . . . .	40

Page

TABLE OF CONTENTS (CONTINUED)  
LIST OF FIGURES

		Page
11(a)	Inverse Transform Simulation with Dimensional Analogy . . . . .	41
11(b)	Direct Transform Simulation with Dimensional Analogy. . . . .	41
12	Butterworth Power Density Spectra . . . . .	45
13(a)	Central Butterworth Correlation Function: First Case. . . . .	46
13(b)	Central Butterworth Correlation Function: Second Case . . . . .	46
13(c)	Central Butterworth Correlation Function: Third Case. . . . .	47
13(d)	Central Butterworth Correlation Function: Fourth Case . . . . .	47
13(e)	Noncentral Butterworth Correlation Function: First Case . . . . .	48
13(f)	Noncentral Butterworth Correlation Function: Second Case. . . . .	48
13(g)	Noncentral Butterworth Correlation Function: Third Case . . . . .	49
13(h)	Noncentral Butterworth Correlation Function: Fourth Case. . . . .	49
14	Preemphasized Baseband Spectrum with Rectangle Input. . . . .	52
15	Equivalent Phase Modulating Spectrum with Rectangle Input . . . . .	52
16(a)	Equivalent Phase Modulating Spectrum with P=50 and P=100 Butterworth Inputs. . . . .	53
16(b)	Equivalent Phase Modulating Spectrum with P=10 and P=20 Butterworth Inputs . . . . .	53
17(a)	FDM/FM Spectrum for m=0.10. . . . .	55
17(b)	FDM/FM Spectrum for m=0.33. . . . .	55
17(c)	FDM/FM Spectrum for m=0.5 . . . . .	56
17(d)	FDM/FM Spectrum for m=1.0 . . . . .	56
17(e)	FDM/FM Spectrum for m=2.0 . . . . .	57
17(f)	FDM/FM Spectrum for m=3.0 . . . . .	57
17(g)	FDM/FM Spectrum for m=4.0 . . . . .	58
17(h)	FDM/FM Spectrum for m=5.0 . . . . .	58
18(a)	FDM/FM Spectrum of 1800-Channel Bell TD2 System . . . . .	59
18(b)	FDM/FM Spectrum of 2400-Channel Bell TH System. . . . .	59
18(c)	FDM/FM Spectrum of 1500-Channel Bell TD2 System . . . . .	60
18(d)	FDM/FM Spectrum of 1500-Channel AT&T System . . . . .	60
18(e)	FDM/FM Spectrum of 1800-Channel AT&T System . . . . .	61
18(f)	FDM/FM Spectrum of 1200-Channel AT&T System . . . . .	61
18(g)	FDM/FM Spectrum of 360-Channel AT&T System. . . . .	62
18(h)	FDM/FM Spectrum of 600-Channel AT&T System. . . . .	62
18(i)	FDM/FM Spectrum of 360-Channel Western Union System . . . . .	63
18(j)	FDM/FM Spectrum of 420-Channel Western Union System . . . . .	63
18(k)	FDM/FM Spectrum of 180-Channel Western Union System . . . . .	64
18(l)	FDM/FM Spectrum of 120-Channel Western Union System . . . . .	64
19	Conversion Factor ( $\beta/m$ ) as a Function of Baseband Parameter ( ) . . . . .	69

## ABSTRACT

This report is concerned with the spectral representation of analog FM signals, with particular attention to FDM/FM satellite communication systems. The FM spectral modeling and gaussian approximation principles are analyzed and extended to develop computer simulation programs capable of providing representative FM spectra. A generalized program is developed to accommodate a variety of baseband and preemphasis characteristics, and adapted to generate FDM/FM telephony spectra. The program features the automatic validation and generation of the gaussian spectrum model if applicable, or the automatic simulation of the modulation process to generate the FM spectrum samples otherwise. The program is used to simulate a collection of satellite FDM/FM telephony spectra, which are to be applied as input data into other available interference analysis programs, as part of a major automated computer capability dedicated to the comprehensive assessment of orbital congestion and spectrum resource management concerns pertinent to national and international satellite communication systems scenarios.

## KEY WORDS

FM Spectrum Models  
Gaussian Spectral Approximation  
FM Spectrum Simulation  
FDM/FM Telephony Spectra

## SECTION 1

### GENERAL INTRODUCTION

The National Telecommunications and Information Administration (NTIA) is responsible for managing the radio spectrum allocated to the U.S. Federal Government. Part of NTIA's responsibility is to: "...establish policies concerning spectrum assignment, allocation and use, and provide the various Departments and agencies with guidance to assure that their conduct of telecommunications activities is consistent with these policies" (Department of Commerce, 1980). In support of these requirements, NTIA performs spectrum resource assessments to identify existing or potential spectrum utilization and compatibility problems among the telecommunication systems of various departments and agencies. NTIA also provides recommendations to resolve any spectrum usage or allocation conflicts, and to improve the spectrum management functions and procedures.

NTIA is engaged in the development of an automated computer capability to be used by the Federal Government for the comprehensive assessment of national and international satellite communication systems. The program will feature both interference evaluation and logical optimization of a varying systems population, thus supporting the orbit and spectrum resource management functions. The effective coexistence of multiple satellite systems and service signal transmissions represents a critical concern from the orbital congestion, communications interference and service reliability standpoints.

The orbital and spectrum congestion introduces unwanted signals into the antennas and receivers of dedicated satellites and earth stations. The interfering signals processed by the satellite transponder and earth station receiver equipment ultimately appear as degradation effects on the desired output information, whether it be analog messages or digital symbols. The link geometries and power budgets of the various satellite systems establish desired and interference signal levels at the receiving station inputs, which need to be converted into output degradation effects so as to guide the logical assessment of the operational scenarios.

The development of receiver transfer characteristics to evaluate the interference degradation effects requires accurate spectral representations of the signals involved (Jeruchim and Kane, 1970; Pontano, et al, 1973; Das and Sharp, 1975). Many existing models and formulations contain simple qualitative assumptions or restricted parametric conditions as validity constraints, with more accurate spectral representations needed to employ the available results or develop new ones as required. For example, the compact formulations available for analog FM applications are conditioned on extreme high or low modulation indices, with representation uncertainties hindering their usage in intermediate index situations.

The sections that follow are concerned with the spectral representation for analog FM applications. The spectral modeling and gaussian approximation principles are first identified in Section 2, and then extended to develop effective FM spectrum simulation programs capable of resolving the modeling concerns and providing representative FM spectra. The programs developed consist of a specific one dedicated to a particular baseband modulation, plus a

generalized one capable of handling a wide variety of modulation characteristics. The specific program presented in Section 3 features the only nontrivial modulation case where a compact formulation results for the output spectra. The program algorithm reproduces the output spectrum formula, thus bypassing the need to simulate the modulation process itself.

The generalized simulation program of Section 4 then accommodates a variety of baseband and preemphasis characteristics with minimal assumptions, by actually simulating the modulation process via equivalent block functions and transform processors. This program was further adapted to produce FDM/FM telephony spectra by including a baseband spectrum driver and CCIR preemphasis, with the high and low baseband frequencies and the rms multichannel frequency deviation selectable by the user. It also features an adjustable bandwidth expansion parameter that accounts for the FM spectral expansion while controlling the distortion and aliasing effects of the discrete representations.

The validity of the gaussian approximation for the FM spectral representation under high modulation index conditions was analyzed using both the specific and generalized FM spectrum programs. A gaussian spectrum generation algorithm was included in each program, and spectral comparisons were performed to identify the modulation index constraints needed for the gaussian spectral approximation to hold. The programs can thus deliver either the simulated FM spectra or their wideband gaussian approximation as needed, and can be used as inputs to other programs dedicated to evaluate receiver transfer characteristics from given spectral representations of the desired and interference signals.

The generalized FM spectrum generation program was employed to generate a collection of FDM/FM telephony spectra representative of existing and planned satellite communication systems. The available system specifications are used to provide the input parameters needed for the spectral generation, and the FDM/FM output spectra resulting from the simulation program are automatically computed and plotted along with the gaussian spectral representation for comparison purposes.

The FDM/FM spectral simulation results are presented in Section 4. The evolution of the gaussian spectral approximation as the modulation index increases is noted to be really governed by the equivalent rms phase deviation parameter, which depends both on the rms modulation index and the low/high frequency ratio of the multichannel baseband modulation. An effective formulation of this dependence is provided in Section 5, and incorporated into the simulation program to automatically trigger the gaussian spectral approximation when valid.

The generalized spectrum simulation program is now operational and automated to deliver the FDM/FM system spectra in an efficient way. The user selects an equivalent set of modulation parameters, and the program first computes the rms phase deviation to decide on the gaussian spectral approximation validity. If the latter is valid, the program next computes the appropriate standard deviation for the gaussian curve from the input parameters, and proceeds to generate the gaussian spectrum samples. Otherwise, the program negates the gaussian logic and proceeds with the FM simulation process to deliver the proper FM spectrum samples.



## SECTION 2

### FM SPECTRAL MODELLING AND GAUSSIAN REPRESENTATIONS

The FM signal spectrum models presently employed only have a compact formulation in certain cases. At low modulation indices, the FM output spectrum is effectively approximated by a discrete carrier component plus a double-sideband continuous spectrum. The latter has the same shape as the equivalent lowpass spectrum that phase modulates the carrier under low index conditions. In particular, such lowpass spectrum will be identical to the input baseband spectrum when ideal FM preemphasis (parabolic power weighting) is employed.

At high modulation indices, the FM output spectrum is characterized by a small discrete carrier component plus a predominant continuous gaussian spectrum centered around the carrier component. The relative power distribution between these discrete and continuous components is uniquely specified by the rms phase deviation. The only other information needed to specify the FM output spectrum is then the gaussian standard deviation or variance parameter, which controls the effective width of the continuous gaussian portion of the spectrum. This parameter has been formulated in terms of the rms phase or frequency deviation employed, and renders the FM spectrum model characterization under high index conditions.

The gaussian spectrum model is assumed to hold regardless of the input baseband spectrum or preemphasis characteristic, as long as the high modulation index exists. However, the identification of what represents a high index condition remains somewhat arbitrary. Also, the variety of baseband spectra, preemphasis characteristics, modulation indices and frequency deviations employed in the different FM signals of interest spans a considerable range of spectral shapes and parameter values, which hinders the spectral approximation evaluation. Hence, the FM spectral modeling issue should be given due attention to assure accurate signal characterizations and permit reliable interference analyses.

Another pertinent issue consists of the parametric value assignment in the gaussian spectrum model. The standard deviation parameter in the gaussian formula is sometimes specified from the rms phase deviation in a PM formulation, and sometimes from the rms frequency deviation in an FM formulation, as discussed in what follows. The conversion is tractable in most baseband cases without preemphasis, but the preemphasized baseband cases can lead to computational difficulties. The preemphasis network can be designed to preserve the rms phase or frequency deviation but not both in general, and the evaluation of the one not being preserved may be difficult yet required if the gaussian spectral representation is to be employed.

### GAUSSIAN SPECTRAL APPROXIMATION PRINCIPLES

The original principle supporting the gaussian spectral approximation under high index conditions is based on Woodward's theorem (Blackman and McAlpine, 1969). It states that the limiting form of the FM power density spectrum as the index increases is given by the probability distribution of the instantaneous